Effect of Wind on Wave Shape: Shallow Water

Thomas Zdyrski [1] Falk Feddersen [1]

[1]Scripps Institution of Oceanography, UC San Diego

Introduction: Wind-Wave Coupling

- Early theoretical investigations (*e.g.*, Jeffreys 1925; Miles 1957; Phillips 1957) into wind-wave coupling focused on deriving growth rates
- Most employed **phase-averaging** technique to extract energy and momentum fluxes
- This removes phase-dependent information such as wave shape

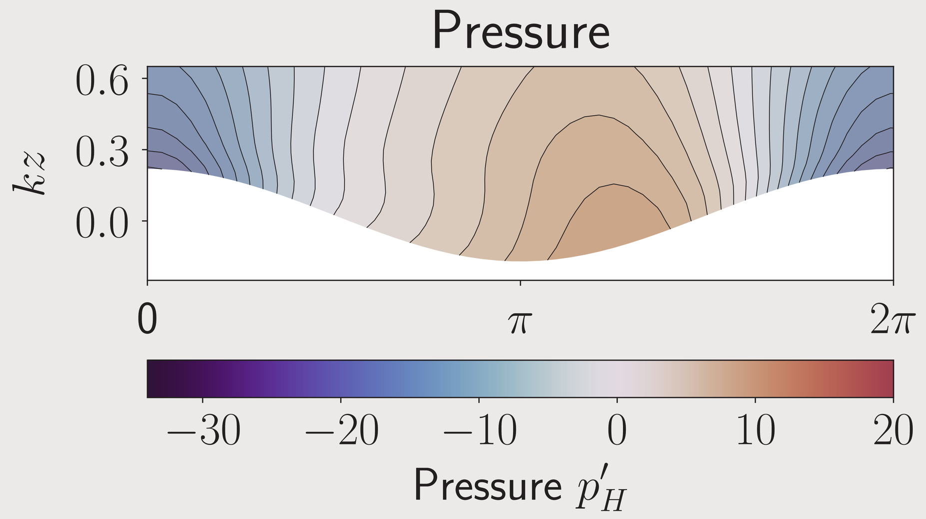

Figure 1: LES of pressure above a wave (Husain et al. 2019)

- **Numerical simulations** have revealed airflow properties above wave fields
- But many prescribe fixed, sinusoidal wave profiles
- Therefore, a **new approach is needed** to relate wind and wave shape

Introduction: Wave Shape

- Wave shape is important in many disciplines, *e.g.*, beach morphodynamics and remote sensing
- A few **laboratory experiments** have quantified wind-speed dependent changes to wave shape

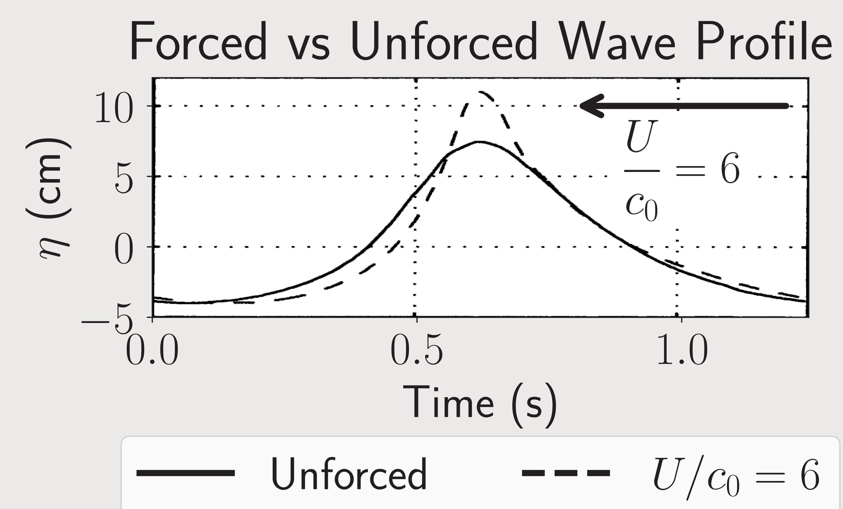

Figure 2: Reproduced from Feddersen and Veron (2005)

- Wind modifies **skewness** S (Cox and Munk 1956) and A **asymmetry** (Leykin et al. 1995)

$$S = \frac{\langle \eta^3 \rangle}{\langle \eta^2 \rangle^{3/2}} \quad \text{and} \quad A = \frac{\langle \mathcal{H}[\eta]^3 \rangle}{\langle \mathcal{H}[\eta]^2 \rangle^{3/2}} \tag{1}$$

- $\langle \cdot \rangle$ is an average over a wave period and $\mathcal{H}$ is the Hilbert transform

Contact Information

- Email: tzdyrski@physics.ucsd.edu
- Web: `physics.ucsd.edu/~tzdyrski`

Introduction: Intermediate/Deep Water (Zdyrski and Feddersen 2019)

- Stokes wave ansatz (with wave amplitude a, wave number k, and phase speed c)

$$\eta k = (ak)\sin[k(x-ct)] + \frac{1}{2}(ak)^2 A_2 \sin[2k(x-ct)+\beta] \tag{2}$$

- **Biphase** β: phase shift between primary and first harmonic (zero for unforced Stokes wave)

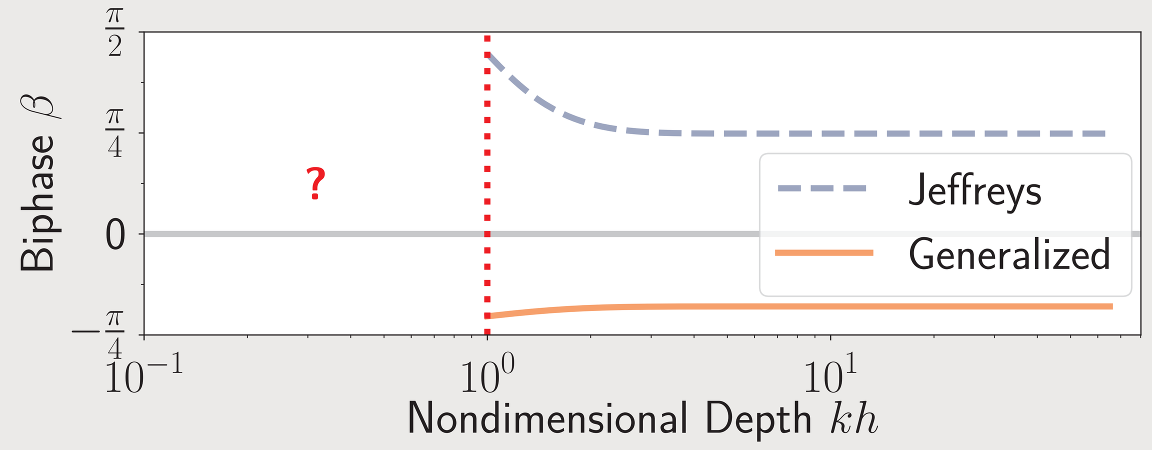

Figure 3: Wind-induced biphase for two forcing types in intermediate/deep water (Zdyrski and Feddersen 2019)

- Decreasing kh amplifies wind-induced shape change, motivating current study for $kh < 1$

Setup: Governing Equations

- Profile $\eta(x,t)$ and potential $\nabla\phi(x,t,z) = \vec{u}$
- Standard incompressibility, bottom boundary, and surface boundary conditions
- Pressure enters Bernoulli equation

$$0 = g\eta + \frac{\partial\phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2\right] + \frac{p}{\rho_w} \quad \text{at} \quad z = \eta \tag{3}$$

- Choose **Jeffreys-type** forcing:

$$p(x,t) = P\partial_x \eta(x,t) \tag{4}$$

Setup: Perturbation Expansion

- Expand dependent variables in small parameter ε

$$\eta = \eta_0 + \varepsilon\eta_1 + \varepsilon^2\eta_2 + \ldots$$

- **Multiple Scales** Method: use slower timescales

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \varepsilon\frac{\partial}{\partial t_1} + \varepsilon^2\frac{\partial}{\partial t_2} + \ldots$$

- Collect orders of ε gives **KdV-Burgers** equation

$$\frac{1}{c_0}\frac{\partial\eta_1}{\partial t_1} + \frac{3\eta_1}{2}\frac{\partial\eta_1}{a\,\partial x} + \frac{1}{6k^2}\frac{\partial^3\eta_1}{\partial x^3} = -\frac{P}{\rho_w g}\frac{1}{2}\frac{\partial^2\eta_1}{\partial x^2} \tag{5}$$

with $c_0 = \sqrt{gh}$

- $P > 0$ for onshore wind, $P < 0$ for offshore

Results: Solitary Profile

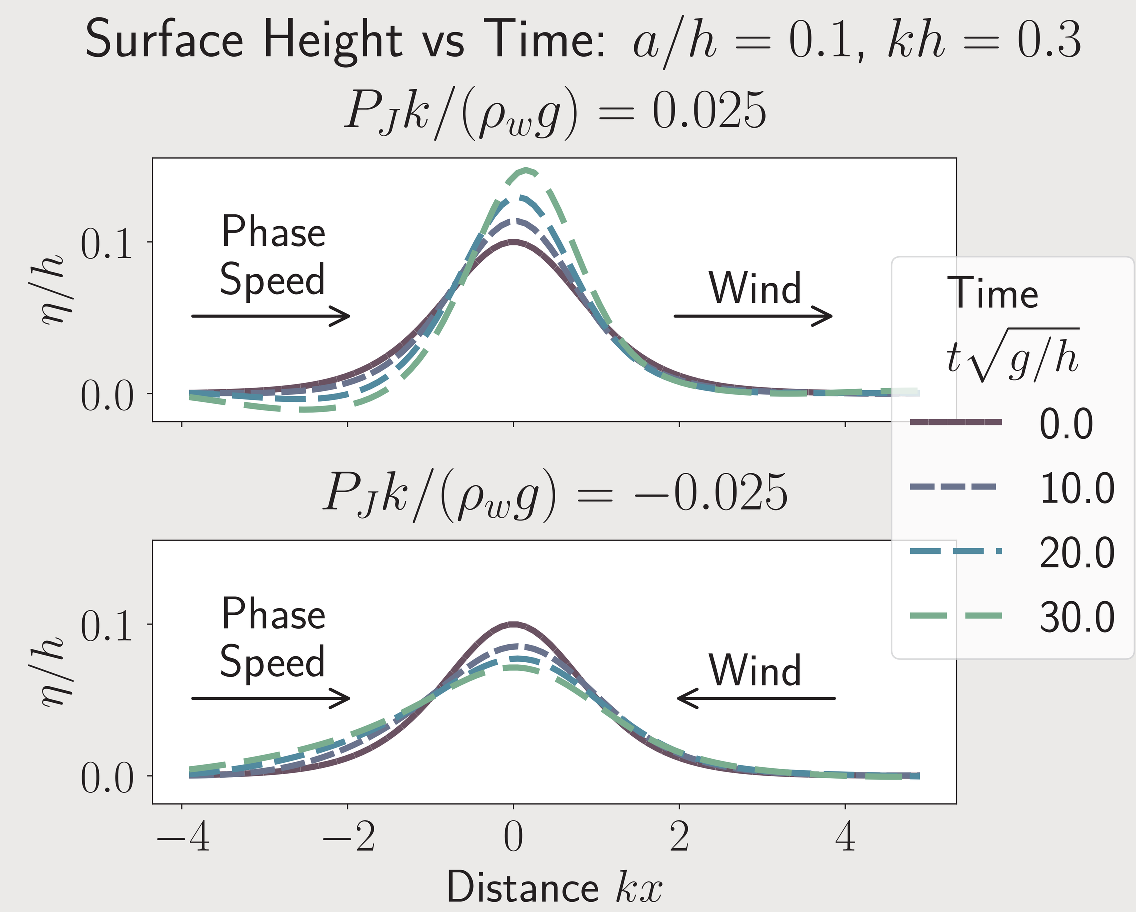

Figure 4: Onshore and offshore winds acting on soliton profiles $k\eta$ as a function of distance kx at various times $t\sqrt{g/h}$, shown in a frame moving with the unforced phase speed

- As time increases, the top plot—depicting onshore wind—shows a **growing and steepening effect**, while the bottom, offshore wind depicts a decaying and broadening effect
- The wind induces **horizontal asymmetry**, particularly apparent in the bottom plot

Results: Cnoidal Profile

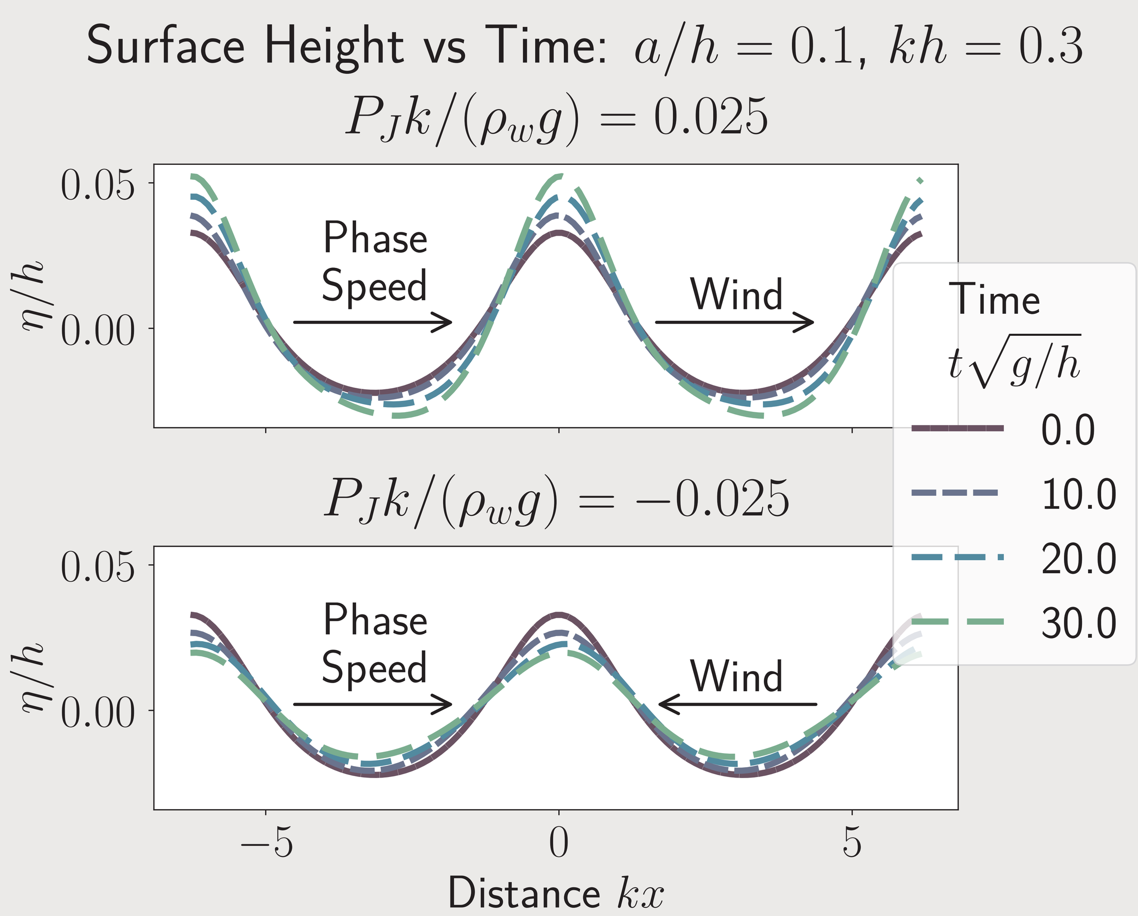

Figure 5: Onshore and offshore winds acting on cnoidal profiles $k\eta$ as a function of distance kx at various times $t\sqrt{g/h}$, shown in a frame moving with the unforced phase speed

- Unforced KdV equation also has periodic "cnoidal" wave solutions
- Onshore wind **increases skewness**; offshore wind decreases it
- Jeffreys onshore (offshore) forcing causes waves to **tilt backwards (forwards)**

Results: Cnoidal Shape Parameters

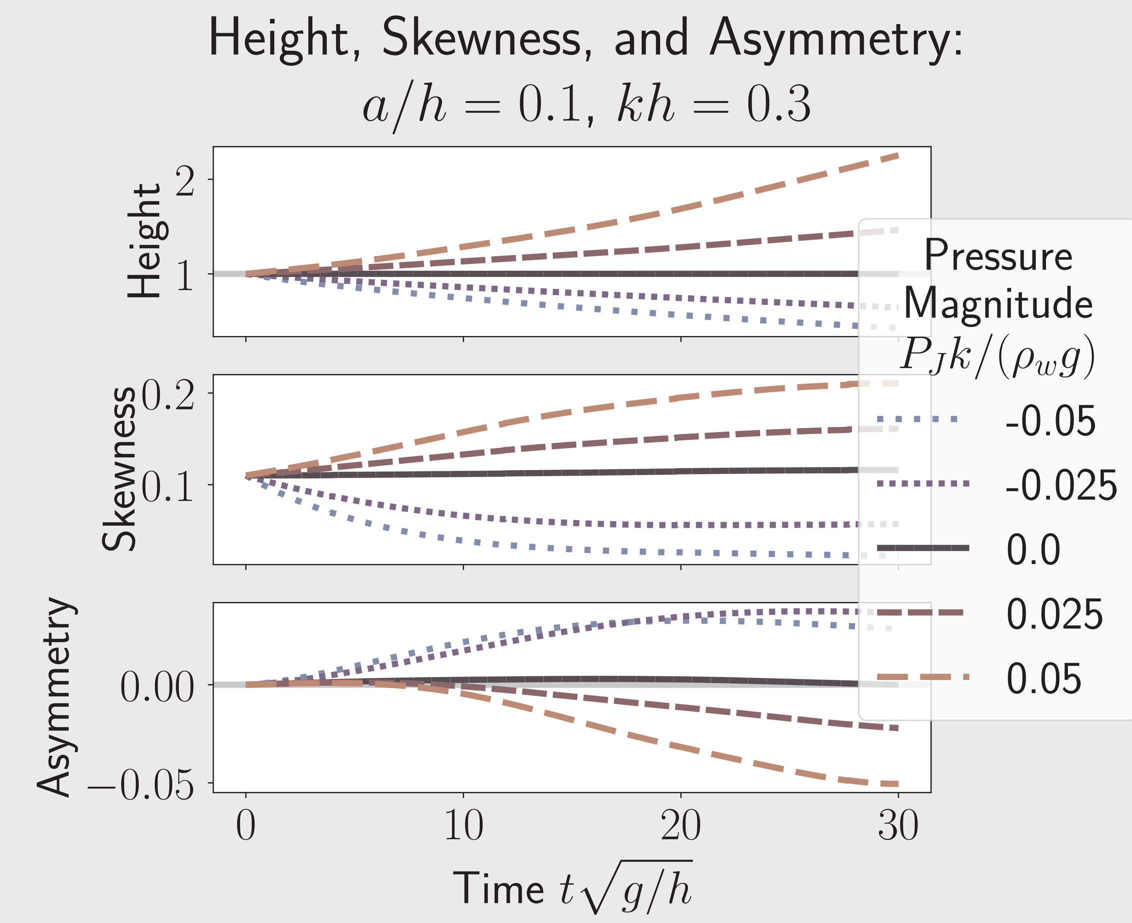

Figure 6: The height (normalized by the initial height), skewness, and asymmetry of cnoidal waves as functions of the nondimensional time $t\sqrt{g/h}$

- **Height, skewness, and asymmetry** given for no wind, onshore wind, and offshore wind
- Onshore breezes yield modest growth, an increasing skewness, and a negative asymmetry
- Offshore winds cause decreasing amplitude and skewness, but increasing asymmetry

Summary

- Coupled surface pressure to the Bernoulli Eq.
- Method of Multiple Scales gave KdV-Burgers Eq.
- Numerically calculated shape changes consistent with casual observations
- **Surface pressure yields appreciable wave shape changes in shallow water**

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Acknowledgements

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